Optimal Paths in Random Resistor Networks

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Abstract

Random resistor networks are often used to model transport in disordered systems, including more complex phenomena such as brittle fracture [1] and the flow of fluids with complex rheologies [2]. This is done by mapping the electrical properties of the individual conductive elements to those of elasto-plastic bars or fluid flow in channels.

A random resistor network is a network of conducting bonds with a piecewise linear characteristic (Fig. 1(a)). As the voltage drop over the bond increase, so does the current, until a threshold is reached. Each bond in the network has a random threshold drawn from a probability distribution, which introduce disorder in the system. This can in turn affect the macroscopic properties of the system. As the voltage drop over the entire network increase, more bonds reach their threshold, and a spanning path of broken bonds eventually forms.

When a spanning path forms, the macroscopic properties of the network change. In the case of fracture, the spanning path is a crack which disconnects the network, while for a Bingham fluid in a porous medium, the fluid flows through the medium along the spanning path. It is of interest where the spanning path forms [2]. It is already known [3] that the spanning path follows the optimal path for a perfect plastic behaviour ($\beta = 0$), where the optimal spanning path is the one which minimize the sum of the thresholds along the path.

We have studied where the spanning paths form in the network in the regime where $0 < \beta < \alpha$ (defined in the figure), and its relation to the optimal spanning path through the system [4]. Using a hierarchial model, we show that the formation of the spanning path is dependent on the ratio of the slopes in the characteristic, $r = \alpha/\beta$. The path should then be identical for different values of α and β, but a constant r, in a given network. As $r \to \infty$, the spanning path should follow the optimal path, both for $\beta = 0$ and $\alpha \to \infty$ with $\beta > 0$. Numerical results support this, and as r increase, the spanning path is more likely to follow the optimal path (Fig. 1(b)). At the same time, the mass of the spanning path decrease towards that of the optimal path (Fig. 1(c)), and the conducting elements of the network act as perfect plastics.

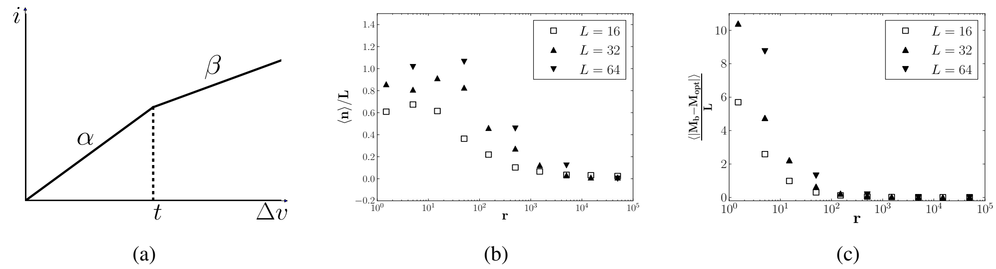

Figure 1: (a) Current-voltage characteristic of a conducting element with a threshold t. α and β denote the slopes of the linear elements. (b) Overlap between optimal path and backbone of spanning path for different r. n denotes the number of bond in the optimal path which are not in the backbone. (c) Difference in the mass of the backbone of the spanning path and the mass of the optimal path for different values of r.

References

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